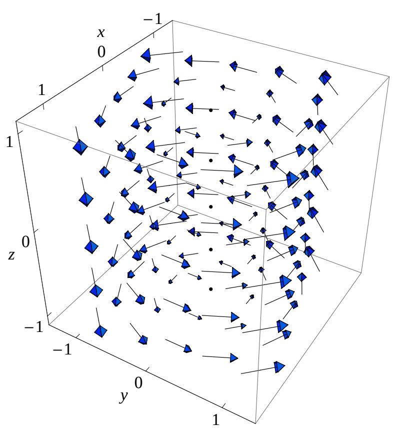
Curl and Divergence

1. (a) $\vec{F}(x,y,z) = \langle y+z, x+2y, x+x^2 \rangle$ is not a conservative vector field. Why not?

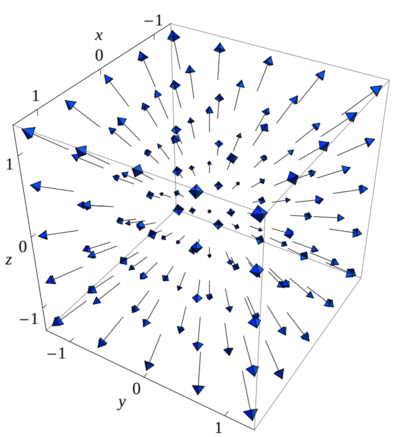
   (b) Let's generalize (a). Let $\vec{F}(x,y,z) = \langle P(x,y,z), Q(x,y,z), R(x,y,z) \rangle$ be a vector field on $\mathbb{R}^3$. If $\vec{F}$ is a conservative vector field, then what must be true about P, Q, and R?

2. Find the curl and divergence of each vector field.

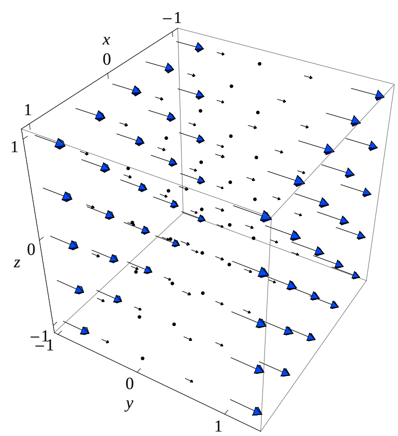
   (a) $\vec{F}(x,y,z) = \langle -y, x, 0 \rangle$



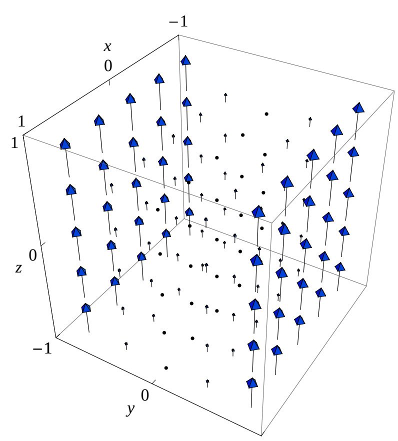
   (b) $\vec{F}(x,y,z) = \langle x, y, z \rangle$



   (c) $\vec{F}(x,y,z) = \langle 0, y^2, 0 \rangle$



   (d) $\vec{F}(x,y,z) = \langle 0, 0, y^2 \rangle$



3. Fill in each blank with either "scalar-valued function of 3 variables" (also sometimes called a "scalar field on $\mathbb{R}^3$") or "vector field on $\mathbb{R}^3$".

   (a) The gradient of a ____ is a ____.

   (b) The curl of a ____ is a ____.

   (c) The divergence of a ____ is a ____.

4. If $\vec{F}(x, y, z) = \langle P(x, y, z), Q(x, y, z), R(x, y, z)\rangle$ is a vector field on $\mathbb{R}^3$, what is $\operatorname{div}(\operatorname{curl} \vec{F})$ in terms of P, Q, and R? How about $\operatorname{curl}(\operatorname{div} \vec{F})$?

5. Suppose the surface of a very small planet is described by the equation $x^2 + y^2 + z^2 = 1$ (where the axes are marked in miles). The population density of green aliens at (x, y, z) is $f(x, y, z) = 10 + x + y + z$ green aliens per square mile. How many green aliens live on the planet? (You may leave your answer as an iterated integral.)